CHAPTER 6.

WHAT IF IT'S *ALL* COMMON KNOWLEDGE? TEACHING ATTRIBUTION PRACTICES IN AN UNDERGRADUATE MATHEMATICS CLASSROOM

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Current writing studies scholarship in attribution practice and instruction is underscored by two central questions: what needs attribution and how should sources and their attributions be incorporated? Professional practice generally answers these questions through systems designed to distinguish authors' original contributions from that of others and from shared/common knowledge in the field. Yet, in STEM classes, and in mathematics in particular, students are often asked to reproduce previously established results and communicate the same thesis and content as their classmates. Consequently, either they have no critical contributions and need to cite everything, or they only present common knowledge and need to cite nothing. Such attribution metrics are thus perplexing rather than clarifying. Using experiences in a mathematics WID classroom, this chapter outlines some challenges of teaching professional attribution strategies through classroom genres that ask students to reproduce common knowledge; it calls for further scholarship to understand and to develop pedagogy to address them.

Within the context of a given community of standards, plagiarism results from authors' failure to distinguish their own contributions sufficiently from the contributions of others. In studying why students plagiarize, Diane Pecorari (2013) distinguished between prototypical *plagiarism*—when writers intentionally deceive others about their original contributions—and *patchwriting*—when writers unintentionally pass off ideas or language as their own because they are unfamiliar with the rhetorical and generic signals of attribution (p. 28). The

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study of prototypical plagiarism revolves around why students cheat whereas the study of patchwriting tends toward questions about the barriers students face when learning to use professional source use practices. Since Rebecca Howard's (1992) eye-opening definition of patchwriting as source-use missteps during the learning process, the study of patchwriting—writing-centered (as opposed to cheating-centered) research into attribution practice—has developed two main strains of questions: What information needs attribution, and how are sources incorporated into a body of writing?

Studies of the latter question are generally interested in what happens when novices try to incorporate sources into their writing. Studies in WID and other upper-level contexts also assume students know they need to use attributions; they are just unsure, unskilled, or unpracticed in the mechanisms for effectively signaling what work is their original contribution and what work is taken from others (cf. Howard & Robillard, 2008; Pecorari & Shaw, 2019). Studies of upper level work (e.g., Jamieson, 2019; Serviss, 2016) tend to focus on this question, addressing upper level students' continued struggles with the means of incorporating others' work rather than what needs any attribution. Additionally, studies like the one in Misty Anne Winzenried's chapter in this collection engage with this line of inquiry; in understanding the geography literature review, the students in her study did not need to determine what needed to be cited but instead had to discover how to distinguish their own argument *about* the literature from their own argument *supported by* the literature. These students therefore needed to learn the rhetorical moves that signaled attributing ideas to sources rather than staking their own claims. As this example suggests, studies in this area of inquiry engage with how students learn the rhetorical techniques they need to distinguish their own contributions from those of others, creating awareness that different communities employ different techniques (see, e.g., Howard & Robillard, 2008).

Studies into the techniques that different communities use to distinguish authors' original contributions are closely related to the former question about what needs attribution. Whether about medium (e.g., Eisner & Vicinus, 2008) or discipline (e.g., Eckel, 2014; Jamieson, 2008), these studies are centered around what kind of information is considered collectively shared information, which can be used without attribution, and what is considered "owned" (Haviland & Mullins, 2009), which needs attribution. Style manuals and handbooks tend to refer to collectively shared information as common knowledge, which Amy England (2008) argued is often implicitly defined in these references as "an established, static set of facts" (p. 109). The shared nature of these facts relates to Kenneth Burke's (1973) parlor metaphor for academic discourse in which the student is a late arrival where everyone else is in the middle of a conversation.

Burke commented that "the discussion had already begun long before any of them [those already in the parlor] got there, so that no one present is qualified to retrace for you all the steps that had gone before" (1973, p. 110). This notion of "no one present [being] qualified" captures the space of what kinds of information pass into the realm of collectively shared knowledge in a community: it needs no attribution because it has lost its source and to attribute credit to anyone in particular is as misleading as attributing it to no one. Yet, students who have not yet been brought into Burke's parlor do not yet share this knowledge with the community and therefore often struggle to distinguish the content considered shared from the content still attributed to particular sources (Shaw & Pecorari, 2019, pp. 5-6). Studies into what needs to be attributed work to clarify such values and develop pedagogies to help introduce new arrivals to the conversation.

Studies in both what and how now attend to discipline- and genre-specific attribution practices, yet their responses coalesce around attribution practices' role in allowing authors to situate their interventions into a community of discussion or a body of knowledge. Distinguishing one's own contribution to a field is generally considered an important component of good academic practice, despite disagreements on what needs to be acknowledged and what forms attributions should take (Pecorari, 2013, p. 31). Such professional practice, however, becomes hard to emulate directly in writing classroom settings, especially in introductory STEM courses in which students are asked to replicate a field's well-known results. In mathematics proof-writing classes in particular, assignments do not generally enable students to express original contributions to the field, as students are primarily asked to re-prove established facts that form the basis of the field. Because attribution is not being used as in the profession, the line determining what does or does not need to be cited can often come across as arbitrary norms used to penalize students. If we ignore how this classroom genre induces perceived arbitrariness, our ignorance can exacerbate the perception of instructors as gatekeepers enforcing arbitrary norms around attribution (cf. Pecorari, 2013) and can impede students' abilities to transfer from classroom forms to professional practice (cf. Russell, 1995). Such assignments thus raise the following question: when solving problems that are already established examples (see Figure 6.1), what counts as common knowledge? And, if it is all common knowledge, how can we use these assignments to effectively teach values attached to professional attribution practices?

To explore these questions more concretely, this chapter considers the specific case of a class I teach at Massachusetts Institute of Technology (MIT), in which these problems acquired particular importance in writing assignment design and instruction. The chapter begins by exploring what counts as common knowledge, reviewing discussions both across the curriculum and specifically in STEM fields. In light of this background, the chapter introduces the mathematics writing classroom, in which the dominant form of argumentation is the formal proof, leading to the role of attribution in this space, especially in light of mathematical attribution practice and WID evaluation of peer review.

Prove the following claims:

(a) $\sum_{i=0}^{n} i^n = \frac{n(n+1)(2n+1)}{6}$.

(b) $(1+x)^k \geq (1+xk)$ for all $x \geq 1$and all positive integers k.

(c) $\sqrt{2}$ is irrational. Furthermore, there exists irrational numbers x and y such that x^y is rational.

Figure 6.1. These are sample writing prompts for an assignment in MIT's Spring 2018 18.200, a communication-intensive discrete math course. Students were asked to prove that the (well-known and well-understood) claims listed above are correct.

As this chapter reflects on an experience, it does not offer data-driven arguments and recommendations, and its strategies are also less generalizable because they rely on field-specific attribution practices. Yet, this anecdotal experience brings attribution in the mathematics classroom into the writing studies conversations around attributions, from which it is currently absent. Moreover, I believe the questions raised both by the challenges and the strategies used to address them can be extended to other fields, particularly in STEM and other content-driven subjects. This chapter thus argues not for particular pedagogy but to recognize the transfer challenges created when we try to teach attribution strategies designed for original contributions through assignments asking students to reproduce common knowledge. Such recognition can lead writing studies to explore more fully the questions that classroom genres raise about the pedagogic goals of attribution instruction and how these goals can translate successfully into the WAC/WID classroom.

ATTRIBUTION AND COMMON KNOWLEDGE ACROSS THE CURRICULUM

Writing studies has recognized that students lack mastery of scholarly and professional attribution practices, and this lack is a primary cause of non-prototypical student plagiarism (see Howard, 1992; Howard & Robillard, 2008; Pecorari, 2013). Recognizing this educational (rather than ethical) challenge in students' source use, anti-plagiarism scholarship has worked to understand barriers to students' initiation into scholarly attribution practice. Such explorations have led some scholars to question whether enough commonality exists across attribution systems to teach generalizable, transferable concepts, with many concluding yes (Hayes et al., 2016; Pecorari, 2013).

However, the most generalized level of agreement—not passing off others' work as one's own—tends to form the basis of institutional plagiarism policies. For many academic and professional organizations, plagiarism means failing to attribute adequately (without defining *adequately*), and includes unacknowledged or unattributed use of another's words, ideas, data, or discoveries (see Table 6.1). For example, MIT defines plagiarism as the "use of another's words, ideas, assertions, data, or figures *[that does] not acknowledge that you have done so* [emphasis added]" (Brennecke, 2018, p. 5). As the italicized predicate emphasizes, the shared definition of plagiarism identifies the underlying problem as claiming the work of others as one's own.

Table 6.1: Plagiarism Definitions from Different Academic Organizations

Organization (field represented)	**Definition**
IEEE (electrical engineering)	"the reuse of someone else's prior processes, results, or words without explicitly acknowledging the original author and source" (IEEE, 2018)
MLA (language & literature)	"presenting another person's ideas, information, expressions, or entire work as one's own" (Modern Language Association, 2016, pp. 6-7)
AMS (mathematics)	"[t]he knowing presentation of another person's mathematical discovery as one's own constitutes plagiarism and is a serious violation of professional ethics" (American Mathematical Society, 2005)
NSF (natural sciences)	"the appropriation of another person's ideas, processes, results, or words without giving appropriate credit" (Fischer, 2011, p. 2)
APA (psychology and social sciences)	"Psychologists do not present portions of another's work or data as their own, even if the other work or data source is cited occasionally" (American Psychological Association, 2017).

In this regard, academics and other professionals do share an understanding of what needs attribution. However, writing studies and applied linguistics have shown we only agree on attribution at this high-level overview, and even this high-level overview quickly breaks down over what constitutes "claiming" and what constitutes "another's work." As early as 2001, Miguel Roig argued that university faculty across disciplines—and even within disciplines—did not share standards for distinguishing paraphrasing from plagiarizing because different fields accepted varying levels of textual appropriation (p. 321). More recently, Rebecca Howard and Amy Robillard (2008) called out many layers of differences in *Pluralizing Plagiarism*, and their contributor Sandra Jamieson (2008) noted that her university committee could only agree to prohibit deliberately passing off another's work as one's own, disagreeing about what counts as information that needs attribution and what mechanics are used to identify it (p. 77). She argued that such challenges result from the fact that disciplinary differences in attribution arise from different acknowledgment values.

For example, studies have shown that researchers in STEM are less concerned about word-for-word matches without quotation than in other fields (Buranen & Stephenson, 2009; Eckel, 2014), "plac[ing] a higher priority on the attribution of ideas than the attribution of words" (Eckel, 2014, p. 2). Such studies suggest that disciplinary distinctions often fall around the values of using one's own words and the importance of quotation; text-centered disciplines tend to value quotation in ways that other research forms do not. Jamieson (2008) pointed out that this difference often leads plagiarism policies based on humanities attribution systems to indict acceptable textual appropriation practice in other fields (pp. 77-78). Given such challenges on the level of faculty, it is not surprising that novices find it difficult to develop intuition about what information is considered usable without attribution and what needs attribution.

Intuition about what is usable without attribution is further complicated by the use of the term *common knowledge* to identify information that does not require attribution. MIT's definition of this term is fairly representative of issues around common knowledge (cf. England, 2008): "information that the average, educated reader would accept as reliable without having to look it up." But MIT adds a caveat: "What may be common knowledge in one culture, nation, academic discipline or peer group may not be common knowledge in another" (Brennecke, 2018, p. 8). Such caveats respond to England's (2008) argument that if common knowledge is introduced as highly contextualized, students will more readily internalize the boundaries in their own field and learn others as they enter new fields (p. 112). So while "the average, educated reader" is still an ambiguous construct, the caveat about the contextualized nature of common knowledge demonstrates attribution scholarship's positive influence, at least in the case of MIT's academic policy and pedagogy.

While these interventions work on the level of professional practice, *classroom genres* present complications beyond the mere process of professionalization.[1] In

1 I define classroom genre by example: A term paper, while it may be related to an academic article, does not have identical needs and conventions to a publishable piece.

particular, many undergraduate classroom environments exist to teach students the common knowledge of the field, and writing classrooms in these fields deliberately ask or expect students to reproduce common knowledge. Such challenges are particularly clear in light of England's (2008) explication of the implicit assumptions in writing manuals' definitions of common knowledge and its association with "*an established, static set of facts*" (p. 109). For instance, in mathematics writing classrooms, students are often asked to prove well-known claims already proved elsewhere. If students have learned that so-called common knowledge does not need to be cited, then they might not see any need for or value of attribution practice in the work they produce for class. Yet instructors want students to learn the value of attribution and to practice its forms of attribution while in these writing classrooms.

The difficulty, though, is that attribution needs in the rhetorical situation of classroom genres differ significantly from those of professional practice. As one example, students are not expected to possess the field's common knowledge being taught in the class, and they therefore are often expected to cite content that might be left unattributed in professional publications. Furthermore, professional attribution practices are based on the assumption that the authors can reasonably situate their interventions as a productive contribution to the field. Students in undergraduate writing classrooms rarely have the opportunity to generate truly original ideas for many reasons, including semester time constraints and access to materials. This disconnect between the content the students are writing up and the functionality of the tools they are being asked to use creates challenges in learning both the value and the practice of attribution, inhibiting the transfer of skills into professional practice. Classroom genres thus raise questions about how we can teach students to understand the values behind professional practice in the constructed conditions of the classroom.

THE MATHEMATICS WRITING CLASSROOM

Undergraduate mathematics classes generally teach students the mathematics discovered over the last several centuries. In particular, course instructors generally assign problems they already can prove. To *prove*, in mathematics, means to create "a logical argument that establishes the truth of a statement beyond any doubt. A proof consists of a finite chain of steps, each one of them a logical consequence of the previous one" (Cupillari, 2005, p. 3). Given that proofs are (typically) already known, students are neither expected nor anticipated to generate original interpretations. While students might follow multiple paths to the same answer, the scope of those paths is highly limited: students are expected to use the tools provided in class to arrive at an identical conclusion, namely the claim they have been asked to prove.

Humanities WAC Prompt:	**Mathematics WID Prompt:**
Using at least two examples from *The Newgate Calendar* in addition to this class' assigned reading, explain what types of evidence could be considered compelling to eighteenth-century readers. You may choose to answer this by considering the evolution—if any—from the presentation of evidence in earlier historical texts to the Neo-Classical texts, or you may choose to focus exclusively on the eighteenth century.	Write a formal expository paper (math article format) that explains the equivalence of the following five forms of the Completeness Property: **Statement [M] (Section 1.6 in Mattuck, 1999).** A bounded, monotone sequence converges. **Statement [N] (Theorem 6.1 in Mattuck, 1999).** Suppose $[a_n,b_n]$ is an infinite sequence of nested intervals, whose lengths tend to 0, i.e., $[b_n - a_n] = 0$. Then there is one and only one number L in all intervals; moreover, $a_n \to L$ and $b_n \to L$ as $n \to \infty$. **Statement [B] (Bolzano-Weierstrass theorem, Theorem 6.3 in Mattuck, 1999).** A bounded sequence has a convergent subsequence. **Statement [C] (Theorem 6.4 in Mattuck, 1999).** A Cauchy sequence converges. **Statement [S] (Theorem 6.5A in Mattuck, 1999).** If a subset of the real numbers is non-empty and bounded above, then it has a supremum.

Figure 6.2a. A prompt from a general education communication-intensive course

Figure 6.2b. A prompt from a communication-intensive mathematics course

Since students use the same tools to arrive at the same old conclusion, such assignments challenge traditional approaches to teaching attribution as a matter of orienting one's original insights within current critical conversations (Buranen & Stephenson, 2009, p. 71). The impracticality of this pedagogic goal becomes evident when comparing mathematics assignment prompts to other communication-intensive course assignments. Figure 6.2 compares a prompt from a general-education, communication-intensive humanities class I taught at Case Western Reserve University (Figure 6.2a) to one from a communication-intensive mathematics class I taught at MIT (Figure 6.2b). The prompt in Figure 6.2a asks students to perform a textual analysis and then draw socio-historical conclusions based on that analysis. The mathematics prompt (Figure 6.2b) asks students to prove the equivalence of five different ways of stating the completeness of the set of real numbers, a fundamental property that gives meaning to claims about limits and their behaviors. Once the equivalence of these statements is proven, mathematicians can use whichever form is more useful to them in any individual proof. To prove equivalence, one must show a connected path from any one of these statements to any of the others.

On a surface level, the math writing prompt (Figure 6.2b) has similar freedoms and constraints as the humanities prompt example (Figure 6.2a). Both prompts articulate basic conditions for acceptable submissions: responses to the humanities prompt need to discuss the assigned theme using two eighteenth-century texts and responses to the math prompt need to provide proofs for a complete path. Additionally, both assignments give the students several degrees of freedom for acceptable responses. For the humanities prompt, students may choose any number of acceptable combinations of primary texts. For the mathematics prompt, the student can choose any fully connected path they want. In both contexts, student responses are influenced by and are likely to reproduce class discussion. Instructors in both classes might therefore expect significant commonalities across submissions.

The significant difference between the nature of responses to these prompts, and by extension the difference between proof-based mathematics writing and writing in other fields, is the degrees of freedom allowed in the expression of ideas. In students' responses to the humanities prompt, an instructor would not expect to read linguistically similar and identically framed essays without direct collusion. However, a mathematics instructor would anticipate a high degree of textual overlap—and might be confused if there were not. As Susanna Epp (2003) explained, "mathematical language is required to be unambiguous, with each grammatical construct having exactly one meaning" (p. 888). Consequently, minor changes in expression can be the difference between a true statement (one that holds without exception) or a false one (one with even a single counterexample). Consider, for instance, the following statement:

$$\text{For all } x \in [0,1], \text{we have } \frac{1}{x} > 1. \tag{1}$$

For most real numbers in the closed interval [0,1], this inequality holds because 1 divided by a number between 0 and 1 (i.e., a fraction) is greater than 1; however, (1) is false because of two edge cases. First, when $x = 1$, the left side of the inequality simplifies to 1, but our statement claims the result should be strictly greater than 1. The difference between "greater than" and "greater than or equal to" is the difference between true and false. Second, when $x = 0$, the left side of the inequality becomes 1/0, which does not exist and therefore has no definable relationship to 1. This simple example demonstrates the importance of precision in mathematical communication, and the arrangement and acknowledgment of quantifiers create this precision. Moreover, such language is used to provide a proof, an argument definitionally "beyond any doubt" (Cupillari, 2005, p. 3), so for the mathematics response, there are multiple correct paths to an answer but not multiple correct outcomes.

Such demands for linguistic precision likely cause mathematicians' different relationship with quotation, paraphrase, and textual appropriation from that typically taught in first-year composition (FYC) classes. This difference arises for two reasons: first, there might be only one (or only a few) correct ways to state a claim, and second, even minor rephrasing might introduce large error into the communication. These limits often lead students to assume that there is only one correct way to write a proof. Couple these (mis)conceptions to their awareness that their writing content is already common knowledge, it becomes easier to understand why it is difficult to teach students in a mathematics writing classroom not only the practice of attribution but also its value.

Additionally, the expected precision of mathematics writing underscores the challenges of applying traditional composition pedagogies in relation to the genre of proof writing. Sarah Bryant, Noreen Lape, and Jennifer Schaefer (2014) critiqued previous work on incorporating writing in mathematics and other quantitative subjects for suggesting composition strategies can be imported without attending to generic features of math writing (pp. 92-93; cf. Bahls, 2012; Sterrett, 1982). Moreover, they persuasively explain their interventions for modifying traditional communication pedagogy to meet the needs of the mathematics classroom. However, neither they, nor any of the sources they critique, make attribution practice a significant part of their discussion.

A potential reason for this absence is that undergraduate mathematics students are expected to be able to discover proofs for themselves using only their course materials. Students in proof-writing classes are not expected to do research in the first-year composition (FYC) sense of going out and finding sources to support one's claims. In MIT's proof-writing classes with explicit WID components, students are still not generally expected to find sources, but they are taught to acknowledge sources, like their textbooks, when they use them. Such citation practices closely follow other fields and styles, such as those taught in FYC courses.

WHEN TO ATTRIBUTE IN MATHEMATICS WRITING

In this regard, students in mathematics classes run the same risks as students in other fields—unless they misinterpret content that needs attribution as common knowledge. For example, while a theorem might be common knowledge, a specific proof of it might not be. However, textbooks often do not distinguish between facts in the field and the author's own interventions, so without additional guidance, students might reasonably expect that the proof strategy is as well-known as the rest of the book contents. From a generalist perspective, such failures to cite might be considered patchwriting, in that students "engag[e] in entry-level manipulation of new ideas and vocabulary" (Howard, 1992, p. 233) without sufficiently making it their own and without acknowledging their source(s). However, mathematical precision can lead students to perceive an author's manner of expression as a technical term—and they are not always wrong. Thus, with textbooks as their primary reading material, these students generally have only seen unattributed write-ups of common knowledge.

Moreover, students are encouraged to collaborate with each other to solve (mathematics) problems. Such collaboration on already-solved problems creates complications for using common knowledge as an attribution metric because not only are students not producing original results, they might be using approaches based on someone else's observations and discoveries. According to the American Mathematical Society (2005), "[t]he knowing presentation of another person's mathematical discovery as one's own constitutes plagiarism and is a serious violation of professional ethics." But what counts as another's mathematical discovery, when one is working collaboratively with classmates to re-prove statements that have been proven for over a century?

Such concerns first came to my attention in my first year as a communication instructor for WID mathematics classes at MIT. At MIT, WID classes pair instructors from specific departments with communication instructors from the Writing, Rhetoric and Professional Communication Program. In Spring 2017, I taught a communication-intensive Real Analysis class. We used two strategies to teach students professional mathematics attribution practice. First, we asked students to acknowledge collaboration: students name their collaborators on their submitted papers. This practice is a modified form of co-authorship; listing collaborators signals contributions at the level of invention but not arrangement. However, this practice does not account for the use of materials other than collaborators' insights.

Fortunately, mathematics as a professional field functions as collaboratively as students in a mathematics classroom, and the profession has already designated attribution practices for the students to follow. Though more mathematicians publish individually than is currently common in experimental STEM fields (Mihaljević-Brandt et al., 2016, pp. 2-3), they still frequently collaborate, even when this doesn't result in co-authorship. Because mathematicians' primary outputs are results (theorems) and validation(s) of those results (proofs), they value crediting the individual responsible for a given theorem or specific approach, so long as the ideas are not yet treated as common knowledge. To that end, they credit important contributions from discussions even when they do not constitute formal collaborations. Examples of such attributions appear in Figure 6.3, including one—example (1)—from an author who won a Fields medal, an analogous award to the Nobel Prize.

(1) "*As Alessio Corti explained to the author*, the fact that the dualizing sheaf of $\sum$ is trivial forces the quotient to be the sheaf of sections of the co-normal bundle of the diagonal in $P^1 \times P^1$"

(Donaldson 2018, p. 7; italics added)

(2) "enables us, with a little more effort, to prove existence of extremals on the sphere, by means of the following "renormalization" approach, *due to Karen Uhlenbeck*"

(Lee & Parker 1987, p. 55; italics added)

(3) "We begin with the following general result *which is kindly taught us by the referee*"

(Oguiso 2013, p. 11; italics added)

(4) "The example of this section is a Markov chain obtained by edge flipping on a line *and was suggested to us by Persi Diaconis*".

(Rhodes & Shilling 2018, p. 17; italics added)

Figure 6.3. Examples from papers published in arXiv

The italicized text in Figure 6.3 calls out how math colleagues acknowledge someone who provided a way of writing a proof. As the page numbers in my in-text citations demonstrate, these comments do not appear in prefatory acknowledgments but in the body of the text. The content surrounding the attributions in examples (3) and (4) in Figure 6.3 indicate that these passages are taken from the main text, not footnotes or endnotes. Viewing (1) and (2) in context will verify that I took those from the body of the papers as well. Such acknowledgments are common practice in mathematics. The examples in Figure 6.3 were kindly provided to me by Heather Macbeth within five hours of my query, indicating that she did not have to dig very far into the arXiv to find such forms of attribution. Her inclusion of example (c) in Figure 6.4 was inspired by this practice.

The text of your assignment must of course be in your own words, and should give appropriate and specific credit, for example:

(a) This proof is adapted from [6, Theorem 4.4].

(b) We recall [Mattuck, Theorem 6.2] that a real number *K* is a cluster point of a sequence if and only if it is the limit of some subsequence.

(c) I learned this argument from Sarah Smith.

Figure 6.4. Attribution instruction and template styles

To teach students such attribution practice, our writing assignment handouts included templates for attribution formatting, as shown in Figure 6.4. The introductory instruction incorporates language related to plagiarism policies to invoke students' prior experience with attribution, as they might have received during an FYC-style course. This introduction calls their attention to the similarities in methods and goals in the mathematics citation styles to those in other fields. The key difference in the mathematics style are related to the practice of numbering core statements (definitions, theorems, lemmas, etc.) for easy reference, as the citation system refers to numbered statements rather than numbered pages.

Figure 6.3 shows examples from papers published on arXiv, an online database housed at Cornell University in which mathematics (and other fields with arXivs) prepublish results. This database was developed to deal with the print publication backlog, allowing for faster dissemination of new information. Additionally, since mathematics gives priority to those who publish first, it creates greater egalitarianism in recognition rate, as mathematicians can post as soon as they have written up publishable results. Results published on arXiv are treated by the mathematics and other arXiv-using communities as credible—though not necessarily peer-reviewed—material.

Figure 6.4 presents the attribution instruction and template styles provided to students in an MIT communication-intensive Real Analysis class handout in Spring 2017. Heather Macbeth authored these model templates. The first number in (a) refers to a hypothetical sixth source in a hypothetical reference list (regardless of genre and medium). In (b), the author's name is used because the hypothetical reference list is alphabetical rather than enumerated.

In Spring 2017, my students tended most often to use templates (a) and (b) in Figure 6.4. This result was intuitively expected, as these forms of in-text citation are familiar from readings across the curriculum and should seem relatively familiar to students who arrive in WID classrooms with attribution experience from FYC-type courses. When students failed to apply attributions of forms (a) and (b) in Figure 6.4, their misunderstanding could easily be read as inaccurate assumptions about what constitutes common knowledge. However, sentences attributing components of one's results to others rarely appears in the body of a paper outside the field of mathematics, which might be a primary reason students struggled to include attributions following template (c) in their papers.

THE CHALLENGE OF PEER REVIEW

Our students' struggles with attributing the sources of their proof strategies was exacerbated through the process of peer review. Just as students do not generally arrive in a mathematics classroom familiar with acknowledging the ideas they learned through collaboration, they do not generally arrive in a mathematics class thinking about learning content from peer review, even if they have prior experience of peer review in FYC classes. Even though humanities professionals' publications sometimes acknowledge insights gained from reviewers or other discussants, in my experience, FYC students are rarely encouraged to make similar acknowledgments when revising term papers after a class peer review process. Moreover, while in FYC classes, the peer review process might provide helpful suggestions to improve the persuasiveness of an argument, the black-and-white nature of correcting information seems to occur most frequently in STEM contexts. Because another student could therefore be responsible for the author's correct result, not acknowledging peers' contributions would violate mathematics attribution values and practice.

In Spring 2017 in the Real Analysis class, we experienced this kind of attribution issue when a student's draft paper—submitted after peer review papers were made available to students—followed an almost identical structure of the review peer's argument, without acknowledging collaboration. From one perspective, this would clearly be plagiarism as defined by practices in mathematics, and potentially designated as cheating per MIT's academic integrity policies. Viewed through the lens of common knowledge, however, this ceases to be a case of malicious cheating and becomes instead a case of ignorance about what counts as others' ideas. It became our priority in the second iteration of the course to provide instruction to help students understand attribution values and practice for mathematics specifically, and in academia in general.

Potential incidents like the one we experienced are hinted at in the writing studies literature on mathematics, as well. For instance, one of Bryant et al.'s (2014) discussions around peer review called out students' abilities to improve their writing through peer observation and comment. The authors quote one of their subjects as noting that "it was extremely useful to see other's [sic] work and learn and share better ways of expressing solutions [emphasis deleted]" (2014, p. 100). The student's intent in "better ways of expressing solutions" remains ambiguous, but the student work I have seen leads me to believe this could refer to borrowing phrasing from other students without attributing the phrase to the peer source. So, while learning mathematical precision and correctness is indeed a benefit of peer review, without proper intervention, it can come at the cost of understanding attribution values and practice in mathematics.

In assessing students' (mis)understandings in relation to peer attribution, we recognized that without formal reflection such as that which Bryant et al. (2014) asked of their students, students might not recognize their content-learning that occurs during the review process. Our instructional team acknowledges the benefits of reflection, but our end goal was not simply to make the learning explicit, but to teach attribution practice. We wanted students to recognize their peers as sources, a value described in communication-intensive math curricula across levels (Day & Frost, 2009, p. 106). In light of this goal (and semester time constraints), we decided to make this implicit process explicit in the peer review assignment itself.

We revised our peer review handout so that it explicitly acknowledged the learning aspects of the review process and indicated ways for students to attribute these unfamiliar sources. New language in the handout, revised by Yu Pan and me, included the following directions:

> Now you have the opportunity to read your classmates' papers answering the question and responding to them. There are two main ways you might respond to them:
>
> (1) as a reader, looking for "new" information
>
> (2) as a writer, looking for ways to improve your own work
>
> . . . Do keep in mind that while stylistic changes are free for sharing (e.g., you like someone's use of sectioning), **if you modify your proofs based on your reviewee's work, you *must* acknowledge them in your paper**.

Students were thus explicitly asked to attend to how they use others' works in advancing their own understanding. Calling the students' attention to this role in their process provided space for them to think through the process of learning content through peer review.

In 2018, this approach was successful in that we had no more (recognized) instances of unacknowledged collaboration in our classroom,[2] and students employed a fuller range of attribution practice. Students more frequently included acknowledgments sections in their papers, thanking their peer reviewers for their contributions to the learning process.[3] Additionally, students would occasionally include remarks along the lines of "this proof was developed in collaboration with [peer]." Such attribution showed that students more fully understood what information is usable without attribution or that which needs attribution in this disciplinary context.

CALL FOR FURTHER RESEARCH

For me, this experience elucidated a specific challenge of using classroom genres to teach professional practices. While we can ask students to write "as if" they are in a professional context, when they don't have professional-level content to use, such pretense becomes even more complicated for student implementation (Wardle 2009, p. 779-781). In particular, in courses where we ask students to re-

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2 As is always the case, there is a chance some work that should have been acknowledged passed by us unattributed but unrecognized as such.

3 Since this is the result of personal experience rather than formalized research, I do not have specific results I can share at this time.

produce common knowledge to help them join that community, writing assignments will not be geared toward pushing students to explore new ground. How, then, can we functionally use these courses and assignments to teach students professional practices built around introducing new information?

Our intervention of calling attention to where and when students learn has had some moderate success in the particular context of this class at MIT. Though motivated by personal experience, and not empirical research, the questions raised are expandable, as they call attention to areas left relatively unexplored in WAC/WID literature. In particular, it would be helpful to have more information about the impact on students from the mismatch between common knowledge contents students are asked to produce and the original contribution genres they are asked to perform. While this case study focused specifically on mathematics to illuminate these issues, it seems likely that other STEM fields would struggle with similar concerns and benefit from this data. This data could help the WID community develop discipline-specific instructional strategies and the WAC community develop generalizable pedagogy around determining what information is usable without attribution or that which needs attribution. This would make attribution instruction more transferable between and across communication contexts. I hope the perspective offered in this chapter helps the WAC/WID community develop better strategies, both in disciplinary and generalized contexts, for teaching students to distinguish between what information is usable without and that which needs attribution.

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